DEPARTMENT OF MATHEMATICS

IISER BHOPAL

PHD PROGRAMME MANUAL (2017)

DEPARTMENT-SPECIFIC GUIDELINES

*Refer to the **PG Manual** for Institute-wide guidelines for the Ph.D. programme. This document consists of guidelines specific to the Mathematics department.*

1. COURSEWORK

Students are required to take three core courses in the first two semesters as described below. These courses cover the bare minimum background required for a student to begin preparatory study in the research areas represented in the department.

First Semester	Second Semester
MTH 601: Algebra I MTH 603: Real Analysis MTH 605: Topology I One Departmental Elective (DE)	At least two DE • A student can take reading courses (4/6 credits) to meet 16 credit requirement in a semester • If a student has cleared the written comprehensive, he/she can take Ph.D. thesis credits (4/8 credits) to meet 16 credit requirement in a semester

Third semester onwards, a student can take DE/Reading course/Ph.D. thesis credits, as per the advise of DPGC Convener/supervisor. Ph.D. Thesis credits can only be taken if a student has cleared the written comprehensive exam.

2. GUIDELINES

1. Assignment of Mentor

Every PhD student will be assigned a mentor upon joining the program. The mentor will provide guidance to the student towards choosing the appropriate and also monitor the student's progress from time-to-time until he/she finds a thesis supervisor.

2. Assignment of Thesis Supervisor

1. Each student is required to choose a potential supervisor by the end of the first year.
2. Once the student passes the written section of the comprehensive exam, the potential supervisor will become the official thesis supervisor.

3. Comprehensive Examination

The comprehensive exam consists of two sections, written and oral.

- **Written section:** This exam will be in one of the three core areas (algebra, analysis, geometry/topology). The written exams will be conducted twice a year (in December and July) by a committee of faculty members appointed by the HoD in consultation with DPGC.

  1. The student should submit Form A to the Department office by April 30 or October 30 to appear for the written comprehensive examination in July or December, respectively. This should be done in consultation with a potential thesis supervisor or DPGC (in case a potential supervisor has not been identified).
  2. The potential supervisor may ask the student to take the written exam in more than one core area.

- **Oral section:** This exam is to be conducted by a committee of at least three faculty members from the department and comprises of a presentation on a topic in an area of student's research interest.

  1. The potential supervisor must be a committee member who should be willing to serve as the official thesis supervisor if the student passes the comprehensive exam.
  2. This exam cannot be taken unless the student has passed the written section.
  3. The student should submit Form B to the Department office atleast 8 weeks before the oral exam alongwith the synopsis prepared in consultation with the potential thesis supervisor. The oral exam can be scheduled anytime subject to availability of the committee.

Students are expected to clear the written comprehensive by the beginning of 3rd semester. Students must successfully complete both sections before the beginning of the fifth semester. At most two attempts are allowed for each section.

- **Additional Remarks:**
  1. The written exam will be of three-hour duration. The syllabi are attached.
  2. The setting of the written exam for any particular area will be done by a committee of three faculty members: instructor of the corresponding course in that academic year and two other faculty members appointed by the Head of the Department in consultation with the DPGC.
  3. The grades for the written exam will be pass or fail and for the oral exam, satisfactory or unsatisfactory.
  4. The oral exam committee is constituted as per Institute's guidelines for constitution of committee for comprehensive examination.
  5. The oral exam is a presentation by the student followed by a question-answer session with the oral exam committee members. The presentation includes a seminar of at least 30 minutes duration and a brief disquisition on the study undertaken towards the oral exam and as notified in the synopsis.

4. Ph.D. Advisory committee

The thesis supervisor, in consultation with the DPGC, will form a Doctoral Advisory Committee (DAC), consisting of the supervisor and at least two other faculty members from the department. The DAC will oversee the progress of the candidate towards completing the requirements for Ph.D. after the comprehensive exam has been successfully cleared.

5. Annual Progress Report

Following successful completion of comprehensive exam requirements, each student is required to submit a yearly report of progress towards the thesis. This report must be submitted to the DAC with a copy to Convener, DPGC.

6. **Graduate Seminar** (referred to as the State-Of-The-Art seminar in the PhD Manual)

- A student is eligible for presenting the Graduate Seminar only after he/she clears the comprehensive examination and has to be completed within 6 months of clearing the comprehensive examination, unless approved by the Chairperson, Senate for any extension. Form C must be submitted atleast one month before the graduate seminar date.
- The performance of the student will only be either Satisfactory or Unsatisfactory; no marks will be awarded. If the performance is evaluated to be the latter, the student has to repeat the Graduate Seminar.
- The Doctoral Advisory Committee (DAC) will evaluate the graduate seminar. The evaluation form will be signed by the supervisor, Convener DPGC and HoD, and will be submitted to the Office of the Academic Affairs.

***FORMS**:*

Form A: Written Comprehensive Examination, Form B: Oral Comprehensive Examination with Synopsis, Form C: Graduate Seminar with DAC *(Other forms are available on Academic Office webpage)*

Reading Courses

Reading courses (MTH 698, MTH 699) are meant for Ph.D. and Integrated Ph.D. students prior to clearing the Ph.D. candidacy requirements, and are in the process of deciding a research topic and a thesis supervisor.

1. MTH 699: Reading Course (6 credits)

• This will be a semester long course.

• There will be one instructor for the course.

• The goal of the course is two fold:
- to promote a focused self-study, and
- to provide the instructor and student the opportunity to work with each other.

• If a student is taking two reading courses in a semester, it must necessarily be with two different instructors.

• A suggested list of reading courses (title, brief outline) with the instructor will be circulated to the students two weeks prior to the pre-registration each semester. The student may contact faculty members for topics outside this list.

• The student should register for the reading course during the pre-registration. The student should submit a hardcopy (Form A) of the title of reading course with instructor's signature, to the Department office, by the pre-registration deadline.

• The student should submit a hardcopy (Form B) of the course contents for the reading course to the department office within a week after the classes begin.

• The student will be required to give two mandatory seminars (for every reading course) in the Department during the course of the semester. The student should submit the seminar details (title and abstract) for each of the seminars to the DPGC no later than one week before the scheduled seminar.

• The course will be evaluated by the instructor based on the performance of the student in the material covered during the course, seminars, and the examinations.

2. MTH 698: Reading Course (with 4 credits)

• The model for this course with 4 credits is the same as for the Reading course (with 6 credits) as described above, except for the seminar component.

PhD Comprehensive Examination Syllabi

1. Algebra

- **Group Theory.** Groups - definitions and examples, Subgroups, Quotient Groups and Homomorphisms, Isomorphism theorems, Group Actions, Direct and Semidirect Products, Finitely generated abelian groups.

- **Rings.** Principal ideal domains and unique factorization domains, Chinese remainder theorem, Noether isomorphism theorems for rings, Noetherian and Artinian rings.

- **Modules.** Definition of Modules, Examples: k[x]-module, abelian group Homomorphisms, Quotient Modules, the four Isomorphism theorems, Direct sums Finitely generated modules, Free modules, Rank of a module, Torsion, Linear independence Free modules over a PID, Finitely generated modules over a PID, Invariant factors, Elementary divisors.

- **Field theory.** Algebraic and transcendental extensions, Finite extensions, degree of a finite extension, Splitting fields of polynomials, Existence and uniqueness of the algebraic closure of a field, Separable and inseparable algebraic extensions, Galois extensions, Galois groups of field extensions, Fundamental theorem of Galois theory, Primitive element theorem, Solvability of polynomial equations by radicals, Examples with computations of Galois groups: Finite fields, cyclotomic extensions.

Suggested Books:

1. Dummit and Foote, *Abstract Algebra*, 2nd edition, Wiley Publications.

2. Hungerford, *Algebra*, Springer Publications.

3. Serge Lang, *Algebra*, 3rd Edition, Addison Welsey.

4. Jacobson, *Basic Algebra*, parts-I and II, 2nd edition, Dover Publications Inc.

5. Birkhoff and McLane, *Algebr*a, Chelsea Publishing Co.

6. D.J.S. Robinson, *A course in the theory of groups*, 2nd edition, Springer Publications.

2. Analysis

Calculus of several variables

- Functions from R^n to R^m, n,m >= 1, continuity and differentiability, directional derivatives total derivative, chain rule, determinants, Jacobian, partition of unity, derivatives of higher order.

- Contraction mapping principle, inverse function theorem, implicit function theorem, rank theorem.

- Riemann integration in R^n, differentiation of integrals, change of variables, Fubini's theorem.

Measure and integration

- Sigma-algebras, Lebesgue measure, measurable functions, Lebesgue integration, modes of convergence, Egoroff's theorem, Fatou's lemma, Lebesgue's monotone convergence theorem, Lebesgue's dominated convergence theorem, Lusin's theorem.

- Product measure, Fubini's theorem, convolution, integration in polar coordinates.

- Signed measures and differentiation, complex measures, total variation, absolute continuity, the Lebesgue-Radon-Nikodym theorem and its applications.

- Lebesgues differentiation theorem, functions of bounded variation, fundamental theorem of calculus for Lebesgue integrals.

- L^p spaces, Holder's inequality, Minkowski inequality, completeness of L^p spaces, convex functions, Jensens inequality, Riesz representation theorem, dual of L^p spaces.

Suggested Books:

1. G.B. Folland, Real analysis: *Modern techniques and their applications*, 2nd Edition, Wiley.

2. J. Munkres, *Elementary Differential topology*, Princeton University Press, 1966

3. H. Royden, *Real Analysis*, 3rd Edition, Prentice-Hall of India, 2008

4. W. Rudin, *Real and Complex Analysis*, 3rd Edition, Tata McGraw-Hill

5. M. Spivak, *Calculus on manifolds*, 5th edition, Westview Press, 1971

6. E. M. Stein and R. Shakarchi, *Real Analysis*: Measure theory, integration, and Hilbert spaces, Princeton lectures in analysis III.

3. Topology/Geometry

- **General Topology:** Continuous functions, Product topology, Metric topology, Quotient Topology, Connectedness, Compactness, Countability, Urysohn's Lemma, Tietze Extension Theorem, Tychonoff Theorem, Local finiteness, Paracompactness, Nagata-Smirnov Metrization Theorem, Compact-open topology, and Ascoli's Theorem.

- **Differentiable manifolds and Lie Groups:** Definition and examples, differentiable functions, existence of partitions of unity, tangent vectors and tangent space at a point, tangent bundle, differential of a smooth map, inverse function theorem, implicit function theorem, immersions, submanifolds, submersions, Sard's theorem, Whitney embedding theorem, Definition of a Lie groups, Matrix groups as examples of Lie groups, and Action of a Lie group on a manifold.

- **Fundamental Groups and Covering Spaces:** Homotopy, Fundamental group of the circle, Fundamental group of a product of spaces, Fundamental group of

the torus, Retractions, Homotopy equivalence, Deformation retractions, Brouwer's Fixed Point Theorem, Borsuk-Ulam Theorem, Covering spaces, Definition of free products and free groups, Universal property of free groups, Van Kampen Theorem, Simple-connectedness of S^n, Deck transformations and isomorphisms of covering spaces, Classification of covering spaces, Covering space actions, and Fundamental groups and covering spaces of some Lie groups.

- **Vector fields and Lie Algebras:** Vector fields, Statement of the existence theorem for ordinary differential equations, One parameter and local one-parameter groups acting on a manifold, The Lie derivative and the Lie algebra of vector fields, distributions and the Frobenius theorem, Definition of of the Lie algebra of a Lie group, The exponential map, Lie subgroups and closed subgroups, and Homogeneous manifolds: definition and examples.

Suggested Books:

1. J. Munkres, *Topology* (2nd Edition), Pearson, 2000.

2. A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.

3. W. Boothby, *An Introduction to differentiable manifolds and Riemannian geometry*, Academic Press, 2002.

4. F. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer, GTM 94, 1983.