

Lecture Series on the Gan-Gross-Prasad Conjectures

by Prof. Dipendra Prasad



Date: June 4, 2024 - June 8, 2024

Venue: AB1-A2

Time: 11.30 am - 12.30 pm

*** Tea will be served at 11.15 am ***

Title:- Around the GGP conjectures on the representation theory of classical groups with emphasis on $GL(n)$

Abstract

An important problem in representation theory is to understand how an irreducible representation of a group G decomposes when restricted to a subgroup H : this is what is called the "branching problem". This question is especially interesting when the multiplicities of representations of H appearing in an irreducible representation of G is bounded by 1, in which case the pair (G,H) is often called a Gelfand pair. This question is studied considerably classically for finite as well as algebraic groups. The papers of Gross-Prasad from 1990's studied such questions for real and p -adic orthogonal groups, as well as for automorphic representations on them, relating the questions to L -functions of interest in number theory. The scope of the Gross-Prasad conjectures was considerably extended in the work of Gan-Gross-Prasad in the last decade, covering all classical groups. The present series of lectures will try to give an exposition of this topic without getting into technical details, emphasizing the group $GL(n)$, and mostly over finite fields. The first lecture will be a colloquium style overview accessible to MSc first year students followed by 4 more lectures which also will be non-technical.

ABSTRACT OF TALKS

Professor Dipendra Prasad

IIT Mumbai

Title: Around the GGP conjectures on the representation theory of Classical groups with emphasis on $GL(n)$.

Abstract: An important problem in representation theory is to understand how an irreducible representation of a group G decomposes when restricted to a subgroup H : this is what is called the “branching problem”. This question is especially interesting when the multiplicities of representations of H appearing in an irreducible representation of G is bounded by 1, in which case the pair (G, H) is often called a Gelfand pair. This question is studied considerably classically for finite as well as algebraic groups. The papers of Gross-Prasad from 1990’s studied such questions for real and p -adic orthogonal groups, as well as for automorphic representations on them, relating the questions to L-functions of interest in number theory. The scope of the Gross-Prasad conjectures was considerably extended in the work of Gan-Gross-Prasad in the last decade, covering all classical groups.

The present series of lectures will try to give an exposition of this topic without getting into technical details, emphasizing the group $GL(n)$, and mostly over finite fields. The first lecture will be a colloquium style overview accessible to M.Sc first year students followed by 4 more lectures which also will be non-technical.

Dr. Venkatasubramanian

IISER Tirupati

Title: Representations of the group $GL(n)$ over finite and p -adic fields.

Abstract: The aim of these elementary talks is to introduce the participants to the smooth representation theory of p -adic groups via representations of the group $GL(n)$ over a non-archimedean local field F . We shall begin by discussing the basic representation theory of the group $GL_n(F)$ where F is a finite field and later move onto the case where F is a p -adic field. We shall introduce the notions of parabolic induction, Jacquet functor and give a coarse classification statement of irreducible representations of $GL_n(F)$. We shall also discuss Bernstein-Zelevinsky’s approach of mirabolic restriction.

Dr. Himanshi Khurana

HRI, Prayagraj

Title: Dimension formula for the twisted Jacquet module of a cuspidal representation of $GL(2n, \mathbb{F}_q)$

Abstract: Let $F = \mathbb{F}_q$ and $G = GL(2n, F)$. Let $P = MN$ be the standard maximal parabolic subgroup of G corresponding to the partition (n, n) . Let π be an irreducible cuspidal representation of G and ψ be any character of N . The ψ -isotypic space of the restriction of the representation π to N is called the twisted Jacquet module of π and is denoted by $\pi_{N, \psi}$. It is an interesting question to understand the structure of $\pi_{N, \psi}$ for any character ψ of N . In this talk, we will discuss the general dimension formula of the twisted Jacquet module $\pi_{N, \psi}$ of π for any character ψ of N . This is a joint work with Dr. Kumar Balasubramanian.

Mohammed Saad Munaf Qadri

IIT Mumbai

Title: Ext branching laws for the general linear group.

Abstract: We study Ext branching laws for Arthur type representations of the p -adic general linear group. We give a precise condition predicting Ext non-vanishing in some cases.

Ankita Parashar

IIT Delhi

Title: Degenerate Whittaker space for GL_4 over length two principal ideal local rings.

Abstract: Whittaker models have been widely used in representation theory in many contexts. The question most studied is describing non-degenerate Whittaker space $\pi_{N,\psi}$ for a representation π of GL_n over finite field by considering a Borel subgroup B with $B = NT$ be its Levi decomposition and a non-degenerate character of N . By a result due to Gel'fand and Graev, one knows that the dimension of non-degenerate Whittaker space for GL_n over finite field is at most one. However, this result no longer holds if we consider a degenerate character of N . In this talk, we describe a degenerate Whittaker space of a regular strongly cuspidal representation of GL_4 over length two principal ideal local rings which is a joint work with Shiv Prakash Patel. We conclude the talk with a conjecture which tells about degenerate Whittaker space of a strongly cuspidal representation of GL_{2n} over length l principal ideal local ring.

Sanjeev Kumar Pandey

IISER Tirupati

Title: Twisted Jacquet modules of representations of symplectic groups.

Abstract: The aim of this talk is to discuss the structure of twisted Jacquet modules of a principal series representation of $Sp_4(F)$. After introducing the notions of parabolic induction and twisted Jacquet functor, we shall describe the explicit structure of the twisted Jacquet module of a parabolically induced representation of $Sp_4(F)$ induced from its maximal parabolic subgroups, where F is a non-Archimedean local field or a finite field. This is a joint work with Venkatasubramanian C. G.

Hariom Sharma

IIT Roorkee

Title: Symplectic model for ladder and unitary representations.

Abstract: In this talk, we classify those ladder representations of $GL_n(D)$, which admit a symplectic model, where D is a quaternion division algebra over a non-Archimedean local field K of characteristic zero. We demonstrate the hereditary nature of the symplectic model for induced representations derived from finite-length representations. Furthermore, we prove a part of Prasad's conjecture, which provides a family of irreducible unitary representations having a symplectic model. This is a joint work with Prof. Mahendra Kumar Verma.

Harshitha

IISER Tirupati

Title: Twisted Jacquet modules of principal series representations of general linear groups.

Abstract: In this talk, we will describe the structure of a twisted Jacquet module of a principal series representation of $GL(2n, F)$ parabolically induced from a maximal parabolic subgroup. The

twisted Jacquet module is taken with respect to a non-degenerate character of the unipotent radical of the standard parabolic subgroup corresponding to the partition (n, n) . Here, F denotes a finite field or a non-Archimedean local field. This is joint work with Venkatasubramanian C. G.

SCHEDULE OF TALKS

June 4, 2024

11.15 am - 11.30 am	Tea
11.30 am - 12.30 pm	Prof. Dipendra Prasad
2.30 pm - 3.30 pm	Dr. C.G. Venketasubramanian
3.30 pm - 4.00 pm	Tea
4.00 pm - 5.00 pm	Mr. Sanjeev Kumar Pandey

June 5, 2024

11.15 am - 11.30 am	Tea
11.30 am - 12.30 pm	Prof. Dipendra Prasad
2.30 pm - 3.30 pm	Dr. C.G. Venketasubramanian
3.30 pm - 4.00 pm	Tea
4.00 pm - 5.00 pm	Dr. Himanshi Khurana

June 6, 2024

11.15 am - 11.30 am	Tea
11.30 am - 12.30 pm	Prof. Dipendra Prasad
2.30 pm - 3.30 pm	Ms. Ankita Parashar
3.30 pm - 4.30 pm	Tea
4.00 pm - 5.00 pm	Mr. Mohammed Saad Munaf Qadri
5.00 pm - 6.00 pm	Mr. Hariom Sharma

June 7, 2024

11.15 am - 11.30 am	Tea
11.30 am - 12.30 pm	Prof. Dipendra Prasad
2.30 pm - 3.30 pm	Ms. Harshitha
3.30 pm - 4.00 pm	Tea
4.00 pm - 5.00 pm	Interaction/Tutorial

June 8, 2024

11.15 am - 11.30 am	Tea
11.30 am - 12.30 pm	Prof. Dipendra Prasad